QCD predictions for the transverse energy flow in deep-inelastic scattering in the HERA small x regime

J. Kwieciński [*], A.D. Martin [†], P.J. Sutton [‡] and K. Golec-Biernat [*]

[*] *Department of Theoretical Physics, H. Niewodniczanski Institute of Nuclear Physics, ul. Radzikowskiego 152, 31-342 Krakow, Poland.*

[†] *Department of Physics, University of Durham, DH1 3LE, England.*

[‡] *Department of Physics, University of Manchester, M13 9PL, England.*

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Abstract

The distribution of transverse energy, E_T, which accompanies deep-inelastic electron-proton scattering at small x, is predicted in the central region away from the current jet and proton remnants. We use BFKL dynamics, which arises from the summation of multiple gluon emissions at small x, to derive an analytic expression for the E_T flow. One interesting feature is an $x^{-\epsilon}$ increase of the E_T distribution with decreasing x, where $\epsilon = (3\alpha_s/\pi)2\log 2$. We perform a numerical study to examine the possibility of using characteristics of the E_T distribution as a means of identifying BFKL dynamics at HERA.

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I. INTRODUCTION

The experiments at HERA measure deep-inelastic electron-proton scattering in the previously unexplored small x regime, $x \lesssim 10^{-3}$. As usual, x is the Bjorken variable $x = Q^2/2p \cdot q$ and $Q^2 = -q^2$, where p and q are the four momenta of the incoming proton and the virtual photon probe respectively. The deep-inelastic observables reflect the small x behaviour of the gluon, which is by far the dominant parton in this kinematic region. In particular the small x behaviour of the structure function $F_2(x, Q^2)$ is driven by the $g \to q\bar{q}$ transition. At such values of x soft gluon emission and the associated virtual gluon corrections give rise to powers of $\log(1/x)$, which have to be resummed. To leading $\alpha_s \log(1/x)$ order, this is carried out by the BFKL (or Lipatov) equation [1] which, in a physical gauge, effectively amounts to the summation of gluon ladder diagrams. Two characteristic features of the result of this procedure are, first, the $x^{-\lambda}$ growth of the gluon distribution $xg(x, Q^2)$ as x decreases, with $\lambda \sim 0.5$, and, second, the relaxation of the strong-ordering of the transverse momenta, k_T, of the gluons along the ladder (which is characteristic of the "large" x, large Q^2 "Altarelli-Parisi" regime).

The recent measurements of $F_2(x, Q^2)$ at HERA [2,3] show a rise with decreasing x which is entirely consistent with the $x^{-\lambda}$ BFKL perturbative QCD prediction [4]. However the observed small x behaviour of F_2 can also be mimicked by conventional dynamics based on the Altarelli-Parisi equation [5]. Because of the inclusive nature of F_2, it is not a sensitive discriminator between BFKL and conventional dynamics. For this purpose it is necessary to look into properties of the final state. Here we investigate whether or not the emitted transverse energy, E_T, which accompanies a deep-inelastic event (see Fig. 1), can be used to identify the BFKL dynamics. Due to the relaxation of the strong-ordering of the gluon k_T's along the chain we expect to find more transverse energy in the central region (between the current jet and the proton remnants) than would result from conventional evolution. Indeed a Monte Carlo study, in the context of heavy quark production, of the E_T distribution as a function of rapidity hints at an increase of the E_T flow if QCD "all-loop" dynamics [6–8] (which incorporates BFKL effects) is employed, see Fig. 9 of Ref. [9].

Now the E_T emitted by the gluon chain is given by the integral over the inclusive distribution for the emission of one gluon weighted by its E_T. Thus, for a given element of (x, Q^2), the distribution of transverse energy as a function of $\log x_j$ is given by

$$x_j \frac{\partial E_T}{\partial x_j} = \frac{1}{\sigma} \int dk_j^2 x_j \frac{\partial \sigma}{\partial x_j \partial k_j^2} |\mathbf{k_j}| \tag{1.1}$$

where σ denotes the cross section in the (x, Q^2) interval, and where x_j and k_j are the longitudinal momentum fraction and the transverse momentum carried by the gluon jet in the photon-proton centre-of-mass frame. The variable $\log x_j$ is closely related to rapidity, see Sec. II B. In terms of the proton structure functions, the differential cross section in Eq. (1.1) is

$$\frac{\partial \sigma}{\partial x_j \partial k_j^2} = \frac{4\pi\alpha^2}{xQ^4} \left[(1-y) \frac{\partial F_2}{\partial x_j \partial k_j^2} + \frac{1}{2} y^2 \frac{\partial (2xF_1)}{\partial x_j \partial k_j^2} \right] \tag{1.2}$$

where $y = Q^2/xs$ with $\sqrt{s}$ being the centre-of-mass energy of the colliding ep system.

With these definitions, we can calculate the energy flow, $x_j \partial E_T / \partial x_j$ of Eq. (1.1), by using the diagrams of Fig. 2 to evaluate the differential structure functions. We have [10–12]

$$x_j \frac{\partial F_i}{\partial x_j \partial k_j^2} = \int \frac{d^2 k_p}{\pi k_p^4} \int \frac{d^2 k_\gamma}{k_\gamma^4} \left(\frac{3\alpha_s}{\pi} \frac{k_p^2 k_\gamma^2}{k_j^2} \right) \mathcal{F}_i(x/x_j, k_\gamma^2, Q^2) f(x_j, k_p^2) \delta^{(2)}(k_j - k_\gamma - k_p) \quad (1.3)$$

with $i = 1, 2$, where $\mathcal{F}_i$ and f describe, respectively the soft gluon resummation above and below the emitted gluon of Fig. 2. That is $\mathcal{F}_i$ and f satisfy BFKL equations, provided of course that x/x_j and x_j are sufficiently small. The BFKL equation for f may be written in the differential form

$$-z \frac{\partial f(z, k^2)}{\partial z} = \frac{3\alpha_s}{\pi} k^2 \int \frac{dk'^2}{k'^2} \left[\frac{f(z, k'^2) - f(z, k^2)}{|k'^2 - k^2|} + \frac{f(z, k^2)}{(4k'^4 + k^4)^{\frac{1}{2}}} \right] \equiv K \otimes f, \quad (1.4)$$

with a similar form [13] for $\mathcal{F}_i$. As usual, f is the unintegrated gluon distribution in the proton, that is with the final integration over dk_p^2 not performed. The expression in brackets in (1.3) arises from the (square of the) BFKL vertex for real gluon emission, where k_p, k_γ and k_j are the gluon transverse momenta in the photon-proton frame.

We may use Eqs (1.1)–(1.3), together with the solutions of the BFKL equations for f and $\mathcal{F}_i$, to determine the E_T flow, $\partial E_T / \partial \log x_j$, which accompanies deep-inelastic scattering, provided $x \ll x_j \ll 1$. Valuable insight can be obtained by using the analytic solutions of the BFKL equation for fixed α_s. Therefore, before presenting the full numerical calculation, we perform in Sec. II an analytic study to establish general characteristics of the E_T distribution. First, we make the simplifying assumption of strong-ordering at the vertex for real gluon emission; in fact in addition to the "normal" strong-ordering, $k_j^2 \gg k_p^2$, in the expression in brackets in Eq. (1.3), we also need to include the contribution from the "anomalous" ordering $k_p^2 \gg k_j^2$. Then we present an exact analytic study by using the BFKL form of the vertex, which incorporates all orderings automatically. In Sec. IV we present the predictions of the E_T distribution obtained when the BFKL equation is solved numerically. At each stage we compare with the E_T flow pattern predicted by coventional dynamics based on the Altarelli-Parisi equation. Section V contains our conclusions.

II. CHARACTERISTICS OF E_T FLOW: AN ANALYTIC STUDY

At first sight the prediction for the shape of the E_T distribution as a function of $\log x_j$ looks simple. The leading small z behaviour of the solutions of the BFKL equation for fixed α_s have the form

$$f(z, k^2) \sim z^{-\lambda}, \qquad \text{with} \quad \lambda = 4\bar{\alpha}_s \log 2 \qquad (2.1)$$

where $\bar{\alpha}_s \equiv 3\alpha_s/\pi$, and similarly for $\mathcal{F}_i$. Thus for x/x_j and x_j sufficiently small, we see that the relevant combination which occurs in Eq. (1.3) has the behaviour

$$\frac{1}{\sigma} \mathcal{F}_i(x/x_j, ...) f(x_j, ..) \sim \frac{1}{x^{-\lambda}} \left(\frac{x}{x_j} \right)^{-\lambda} (x_j)^{-\lambda} \qquad (2.2)$$

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which appears to give an E_T distribution which is independent of x_j, and also of x. Thus well away from both the current jet and the proton remnants, we anticipate a plateau distribution of E_T as a function of $\log x_j$, which remains unchanged as x varies.

However the situation is more subtle. We have overlooked the x and x_j dependences which arise from the integrations over the transverse momenta. To see this we need to look more closely at the analytic solution of the BFKL equation. Recall that the solution may be obtained by taking the Mellin transform $\tilde{f}$ of f:

$$f(z, k^2) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} (k^2)^\omega \tilde{f}(z, \omega) d\omega \tag{2.3}$$

$$\tilde{f}(z, \omega) = \int_0^\infty (k^2)^{-\omega-1} f(z, k^2) dk^2 \tag{2.4}$$

with $c = \frac{1}{2}$. If we substitute Eq. (2.4) into (1.4) we obtain

$$-z \frac{\partial \tilde{f}(z, \omega)}{\partial x} = \tilde{K}(\omega) \tilde{f}(z, \omega) \tag{2.5}$$

where $\tilde{K}$, the transform of the BFKL kernel, is

$$\tilde{K}(\omega) = \bar{\alpha}_s [2\Psi(1) - \Psi(\omega) - \Psi(1-\omega)] \tag{2.6}$$

with $\Psi(\omega) \equiv \Gamma'(\omega)/\Gamma(\omega)$. On substituting the solution of Eq. (2.5) into Eq. (2.3) we have

$$f(z, k^2) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} d\omega (k^2)^\omega \tilde{f}(1, \omega) z^{-\tilde{K}(\omega)}. \tag{2.7}$$

As ω varies along the contour of integration, the maximum value of $\tilde{K}(\omega)$ is found to occur at $\omega = \frac{1}{2}$. The region $\omega \sim \frac{1}{2}$ therefore dominates the small z behaviour of f. To obtain the leading behaviour we expand the various terms in Eq. (2.7) about the point $\omega = \frac{1}{2}$ and find

$$f(z, k^2) \approx z^{-\lambda} \left(\frac{k^2}{k_0^2}\right)^{\frac{1}{2}} \frac{\bar{f}_0(\frac{1}{2})}{[2\pi \lambda'' \log(1/z)]^{\frac{1}{2}}} \exp\left(\frac{-\log^2(k^2/\bar{k}^2)}{2\lambda'' \log(1/z)}\right), \tag{2.8}$$

with an analogous relation for the $\mathcal{F}_i$

$$\mathcal{F}_i(z, k^2, Q^2) \approx z^{-\lambda} \left(\frac{k^2}{Q^2}\right)^{\frac{1}{2}} Q^2 \frac{\bar{\mathcal{F}}_i^{(0)}(\frac{1}{2})}{[2\pi \lambda'' \log(1/z)]^{\frac{1}{2}}} \exp\left(\frac{-\log^2(k^2/aQ^2)}{2\lambda'' \log(1/z)}\right) \tag{2.9}$$

where $\bar{f}_0(\frac{1}{2}), \bar{\mathcal{F}}_i^{(0)}(\frac{1}{2}), k_0^2, \bar{k}^2$ and a are given in terms of the boundary conditions. The quantities λ and λ'' arise from the expansion of $\tilde{K}(\omega)$ of Eq. (2.6) about the point $\omega = \frac{1}{2}$:

$$\tilde{K}(\omega) = \lambda + \frac{1}{2}\lambda''(\omega - \frac{1}{2})^2 + ... \tag{2.10}$$

with λ given by Eq. (2.1) and $\lambda'' = \bar{\alpha}_s 28\zeta(3)$, where the Riemann zeta function $\zeta(3) \approx 1.2$.

Note that the scale of the $\mathcal{F}_i$ is provided by Q^2 and also that, with our definition [13], the $\mathcal{F}_i$ have the dimensions of Q^2. In Eq. (2.8) we have used a slightly different notation for

the function $\tilde{f}(1, \omega)$ at the boundary (which for simplicity we take to be $z = 1$). We have introduced a dimensionless boundary function $\bar{f}_0$ defined by

$$\tilde{f}(1, \omega) = (k_0^2)^{-\omega} \bar{f}_0(\omega). \tag{2.11}$$

Similarly for the $\mathcal{F}_i$ we have introduced dimensionless boundary functions $\bar{\mathcal{F}}_i^{(0)}$ defined by

$$\tilde{\mathcal{F}}_i(1, \omega, Q^2) = (Q^2)^{1-\omega} \bar{\mathcal{F}}_i^{(0)}(\omega). \tag{2.12}$$

To estimate the characteristics of the E_T flow we may omit the longitudinal structure function and assume $2xF_1 = F_2$. It is straightforward to include the small correction arising from $F_L = 2xF_1 - F_2$. Thus (1.1) becomes

$$x_j \frac{\partial E_T}{\partial x_j} = \frac{1}{F_2} \int dk_j^2 x_j \frac{\partial F_2}{\partial x_j \partial k_j^2} |\mathbf{k_j}|, \tag{2.13}$$

where the integrand is specified by (1.3) with f and $\mathcal{F}_2$ given by Eq. (2.8) and Eq. (2.9) respectively, for sufficiently small x_j and x/x_j. Thus to determine the E_T flow we must perform the integrations dk_j^2, d^2k_p, d^2k_γ over the transverse momenta, subject of course to the conservation of transverse momentum as embodied in the delta function in (1.3).

A. Simplified analytic treatment

We can obtain valuable insight by making the assumption that the transverse momenta are strongly-ordered at the vertex for real gluon emission. At first sight, we would expect that this meant just keeping the contribution with $k_j^2 \gg k_p^2$ in Fig. 2. However for the E_T weighted distribution we shall find that there is an equally big contribution coming from the region with "anomalous" ordering $k_p^2 \gg k_j^2$. We evaluate these two contributions in turn.

For the "normal" strong-ordered case with $k_j^2 \gg k_p^2$ we have $k_\gamma^2 \approx k_j^2$; then Eq. (1.3) simplifies, since the d^2k_p/k_p^2 integration over f just gives the integrated gluon distribution $x_j g(x_j, k_j^2)$. In fact in this approximation we recover the formula for deep-inelastic + energetic jet events

$$x_j \frac{\partial F_2}{\partial x_j \partial k_j^2} = \left(\frac{\bar{\alpha}_s}{k_j^4} \right) x_j g(x_j, k_j^2) \mathcal{F}_2(x/x_j, k_j^2, Q^2), \tag{2.14}$$

see, for example, Eq. (2.4) of Ref. [13]. Using the explicit solutions (2.8) and (2.9), and inserting (2.14) into (2.13), we obtain

$$x_j \frac{\partial E_T^{(a)}}{\partial x_j} = \frac{2\bar{\alpha}_s}{F_2} x_j^{-\lambda} \left(\frac{x}{x_j} \right)^{-\lambda} \left(\frac{Q^2 \bar{k}^2}{k_0^2} \right)^{\frac{1}{2}} \frac{\bar{f}_0(\frac{1}{2}) \bar{\mathcal{F}}_2^{(0)}(\frac{1}{2}) I(x, x_j, Q^2)}{[2\pi\lambda'' \log(1/x_j) 2\pi\lambda'' \log(x_j/x)]^{\frac{1}{2}}} \tag{2.15}$$

where

$$I = \int_0^\infty \frac{dk_j^2}{k_j^2} \frac{|\mathbf{k_j}|}{(\bar{k}^2)^{\frac{1}{2}}} \exp\left(-\frac{\log^2(k_j^2/\bar{k}^2)}{2\lambda'' \log(1/x_j)} - \frac{\log^2(k_j^2/aQ^2)}{2\lambda'' \log(x_j/x)} \right). \tag{2.16}$$

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The factor 2 in Eq. (2.15) arises when we translate formula (2.8) for $f(x_j, k_p^2)$ into one for $x_j g(x_j, k_j^2)$. This factor can be inferred by using Mellin transform techniques and noting that the main contribution occurs when $1/\omega \approx 2$.

The other contribution, which we denote $x_j \partial E_T^{(b)} / \partial x_j$, coming from the "anomalous" ordered region $k_p^2 \gg k_j^2$, can be readily shown to be equal to $x_j \partial E_T^{(a)} / \partial x_j$. To see this, we note that for this case $k_\gamma^2 \approx k_p^2$ and so we have

$$x_j \frac{\partial E_T^{(b)}}{\partial x_j} = \frac{\bar{\alpha}_s}{F_2} \int \frac{dk_p^2}{(k_p^2)^2} 2(k_p^2)^{\frac{1}{2}} \mathcal{F}_2(x/x_j, k_p^2, Q^2) f(x, k_p^2),$$

where the factor $2(k_p^2)^{\frac{1}{2}}$ arises from

$$\int_0^{k_p^2} (k_j^2)^{-\frac{1}{2}} dk_j^2 = 2(k_p^2)^{\frac{1}{2}}.$$

If we insert Eqs (2.8) and (2.9) then we find $x_j \partial E_T^{(b)} / \partial x_j$ is also given by Eq. (2.15). This so-called anomalous contribution [14] comes from ordering of transverse momenta that are opposite to that in the ordinary Altarelli-Parisi evolution of the structure function. We have found that in the small x limit and for fixed (albeit large) Q^2, that it gives a contribution to the E_T flow equal to that arising from the normal strong-ordering. Why does this "anomalous" contribution not occur, for instance, in the conventional double leading logarithm (DLL) limit of, say, $F_2(x, Q^2)$? In the total cross section (or in an inclusive structure function) the dominant or leading "anomalous" contribution is cancelled by the virtual corrections which lead to gluon reggeisation [1,11] (or, equivalently, to the "non-Sudakov form factor" of Ref. [6–8]). The cancellation ensures that, at small x, we do not encounter the double logarithmic terms of the form $\alpha_s \log^2(1/x)$. Moreover the virtual corrections guarantee that the conventional DLL terms $\alpha_s \log(1/x) \log Q^2$ only come from the usual strongly-ordered region of transverse momenta, i.e. $k_p^2 \ll k_j^2 \ll Q^2$. It may also be interesting to observe that, unlike the unweighted distributions, the E_T flow is an infrared finite quantity since the factor of $|\mathbf{k}_j|$ in the integrand of (1.1) removes the unintegrable singularity which would otherwise occur at $k_j^2 = 0$, see (1.3). After this aside, we now return to complete the analytic calculation of the E_T flow.

The integral I of Eq. (2.16) can readily be evaluated if we change the variable from k_j^2 to $t = \log(k_j^2/\bar{k}^2)$. Then Eq. (2.16) becomes

$$I = \int_{-\infty}^{\infty} dt \, \exp\left(\frac{t}{2} - \frac{t^2}{2\lambda'' \log(1/x_j)} - \frac{(t-T)^2}{2\lambda'' \log(x_j/x)} \right) \tag{2.17}$$

where $T \equiv \log(aQ^2/\bar{k}^2)$. If we evaluate this Gaussian integral, and insert the result into Eq. (2.15) and into the identical expression for $x_j \, \partial E_T^{(b)} / \partial x_j$, we finally obtain the following analytic prediction for the E_T flow

$$x_j \frac{\partial E_T}{\partial x_j} = x_j \frac{\partial E_T^{(a)}}{\partial x_j} + x_j \frac{\partial E_T^{(b)}}{\partial x_j} = 4\bar{\alpha}_s (Q^2 \bar{k}^2)^{\frac{1}{4}} x^{-\epsilon} \exp\left(-\frac{\epsilon[\log(x_j^2/x) + 4T/\lambda'']^2}{\log(1/x)} \right), \tag{2.18}$$

provided x/x_j and x_j are sufficiently small, where $\epsilon = \lambda''/32$.

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To obtain Eq. (2.18) we have used the BFKL prediction for F_2 in the leading $\log 1/x$ approximation [4]

$$F_2(x, Q^2) \;=\; \int \frac{dk^2}{k^4} f(x, k^2) \mathcal{F}_2^{(0)}(k^2, Q^2) \tag{2.19}$$

where $\mathcal{F}_2^{(0)}$ is the quark box (and crossed box) contribution shown in Fig. 2(b), and where the integration over the gluon longitudinal momentum has been performed. If we insert expression (2.8) for f and we take the small x limit (where the Gaussian factor may be neglected), then the integral becomes proportional to the Mellin transform $\tilde{\mathcal{F}}_2^{(0)}$ at $\omega = \frac{1}{2}$. Using Eq. (2.12) we obtain

$$F_2 \;=\; x^{-\lambda} \left(\frac{Q^2}{k_0^2} \right)^{\frac{1}{2}} \frac{\bar{f}_0(\frac{1}{2}) \bar{\mathcal{F}}_2^{(0)}(\frac{1}{2})}{[2\pi\lambda'' \log(1/x)]^{\frac{1}{2}}}. \tag{2.20}$$

This result for F_2 has been inserted into Eq. (2.15).

Equation (2.18) for the E_T flow applies in the central region away from the current jet and the proton remnants, that is when $x_j/x \gg 1$ and $1/x_j \gg 1$. Contrary to our initial expectation, (2.2), the expression (2.18) depends on x, and also on x_j. The main characteristics of the E_T flow can be readily identified from Eq. (2.18). We see that the E_T distribution has a broad Gaussian shape in $\log x_j$ (or in rapidity, see Sec. II B below) with

(i) a peak which moves away from $x_j \sim x^{\frac{1}{2}}$ as Q^2 increases,

(ii) a width which grows as $\sqrt{\log(1/x)}$ as x decreases,

(iii) an $x^{-\epsilon}$ dependence such that it grows with decreasing x,

(iv) a $(Q^2)^{\frac{1}{4}}$ dependence such that it grows with increasing Q^2.

Although the shape of the distribution is specified, the normalisation is controlled by $\log(\bar{k}^2)$, the parameter which gives the centre of the input (non-perturbative) $\log k^2$ Gaussian distribution of f, see Eq. (2.8) and Ref. [4].

B. E_T flow as a function of rapidity

We have calculated the E_T flow as a function of $\log x_j$, a variable which is closely related to the rapidity, y. The result, $\partial E_T / \partial \log x_j$, is the Gaussian form of Eq. (2.18). For practical purposes the rapidity distribution $\partial E_T / \partial y$ is more useful, so here we show how to evaluate the integral I of Eq. (2.17) in terms of y, rather than $\log x_j$. In this way we will find that the E_T flow as a function of rapidity also has a Gaussian form.

We define the rapidity y in the photon-proton c.m. frame in the standard way

$$y \;=\; \tfrac{1}{2} \log \left(\frac{x_j^2}{x} \frac{Q^2}{k_j^2} \right), \tag{2.21}$$

which leads to the following relation between y and $\log x_j$

$$\log x_j \simeq y - \tfrac{1}{2}(T-t) - \tfrac{1}{2}\log(1/x). \tag{2.22}$$

In terms of y, the integral I of Eq. (2.17) assumes the (non-Gaussian) form

$$I = \int_{t_1}^{t_2} dt \, \exp\left(\frac{t}{2} - \frac{t^2}{\lambda''[-2y+T-t+\log(1/x)]} - \frac{(t-T)^2}{\lambda''[2y-T+t+\log(1/x)]}\right) \tag{2.23}$$

where the limits

$$t_{1,2} = T - 2y \mp \log(1/x). \tag{2.24}$$

Now, to obtain an analytic form we approximate the t factors in the denominators in Eq. (2.23) by

$$\bar{t} = 4\epsilon \, \log(1/x) + \tfrac{1}{2}T,$$

which denotes the value of t for which the integrand in Eq. (2.17) is a maximum in the leading $\log(1/x)$ approximation. It is straightforward to evaluate the integral approximated in this way. We find

$$\frac{\partial E_T}{\partial y} = 2\bar{\alpha}_s(Q^2\bar{k}^2)^{\frac{1}{4}}x^{-\epsilon} \exp\left(\frac{-\epsilon[2y+4T/\lambda'' - \tfrac{1}{2}T + \tfrac{1}{8}\lambda''\log(1/x)]^2}{\log(1/x)}\right). \tag{2.25}$$

Thus we obtain a Gaussian E_T distribution as a function of rapidity y, identical in structure to that in terms of $\log x_j$, except for the two non-leading terms in the exponential.

C. Improved analytic treatment

Equation (2.18), though valuable to see the origin of the x and x_j dependence of E_T, is based on assumptions which are not necessary even for an analytic treatment. In this subsection we show that it is not necessary to assume (i) strong-ordering at the gluon emission vertex and (ii) the approximate Gaussian diffusion patterns of Eq. (2.8) and Eq. (2.9). The procedure is to evaluate (1.1)–(1.3) using Mellin transform techniques. We have

$$x_j\frac{\partial E_T}{\partial x_j} = \frac{1}{F_2}\int \frac{dk_p^2}{k_p^2}\int \frac{dk_\gamma^2}{k_\gamma^2}\int \frac{d\phi}{2\pi}\frac{\bar{\alpha}_s}{(k_p^2+k_\gamma^2+2k_pk_\gamma\cos\phi)^{\frac{1}{2}}}\mathcal{F}_2(x/x_j, k_\gamma^2, Q^2)f(x_j, k_p^2)$$

$$\equiv \frac{1}{F_2}\int \frac{dk_p^2}{k_p^2}\int \frac{dk_\gamma^2}{k_\gamma^2}\frac{1}{(k_p^2k_\gamma^2)^{\frac{1}{4}}}H(k_\gamma^2/k_p^2)\mathcal{F}_2(x/x_j, k_\gamma^2, Q^2)f(x_j, k_p^2). \tag{2.26}$$

The function H, introduced as above, is now replaced by the inverse Mellin transform

$$H(k_\gamma^2/k_p^2) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} d\nu \, \tilde{H}(\nu)\left(\frac{k_\gamma^2}{k_p^2}\right)^{\nu} \tag{2.27}$$

with c in the range $-\tfrac{1}{4} < c < \tfrac{1}{4}$. Proceeding in this way we see that the k_p^2 and k_γ^2 integrations, over f and $\mathcal{F}_2$ respectively, now factorize and, moreover, are simply of the form of the Mellin transforms $\tilde{f}$ and $\tilde{\mathcal{F}}_2$, defined as in Eq. (2.4). Thus Eq. (2.26) becomes

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$$x_j \frac{\partial E_T}{\partial x_j} = \frac{1}{F_2} \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} d\nu \, \tilde{H}(\nu) \tilde{\mathcal{F}}_2(x/x_j, \tfrac{1}{4} - \nu, Q^2) \tilde{f}(x_j, \tfrac{1}{4} + \nu). \tag{2.28}$$

The Mellin transform $\tilde{f}(z, \omega)$ satisfies the transformed BFKL equation (2.5) which has the solution

$$\tilde{f}(z, \omega) = (k_0^2)^{-\omega} \bar{f}_0(\omega) z^{-\tilde{K}(\omega)}, \tag{2.29}$$

see Eq. (2.11). Similarly the solution of the equation for $\tilde{\mathcal{F}}_2$ is

$$\tilde{\mathcal{F}}_2(z, \omega, Q^2) = (Q^2)^{1-\omega} \bar{\mathcal{F}}_2^{(0)}(\omega) z^{-\tilde{K}(\omega)}, \tag{2.30}$$

recall Eq. (2.12). The explicit form of the function $\bar{\mathcal{F}}_2^{(0)}(\omega)$ can be simply deduced from, for example, Appendix B of Ref. [13]. Inserting solutions Eq. (2.29) and Eq. (2.30) into Eq. (2.28), we obtain

$$x_j \frac{\partial E_T}{\partial x_j} = \frac{1}{F_2} \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} d\nu \, \tilde{H}(\nu) \frac{(Q^2)^{\frac{3}{4}+\nu}}{(k_0^2)^{\frac{1}{4}+\nu}} \bar{\mathcal{F}}_2^{(0)}(\tfrac{1}{4} - \nu) \bar{f}_0(\tfrac{1}{4} + \nu) \left(\frac{x_j}{x} \right)^{\tilde{K}(\frac{1}{4}-\nu)} \left(\frac{1}{x_j} \right)^{\tilde{K}(\frac{1}{4}+\nu)}. \tag{2.31}$$

We see that in the limit where $x_j/x \gg 1$ and $1/x_j \gg 1$, and in particular in the central region away from the current jet and the proton remnants where $x_j \sim x^{\frac{1}{2}}$ (or to be more precise where $\log(1/x_j) \sim \log(1/x^{\frac{1}{2}})$), that the integral is dominated by contributions from the region with $\nu \sim 0$. We therefore expand the relevant quantities about $\nu = 0$ and evaluate the resulting Gaussian integral. We find the master analytic formula

$$x_j \frac{\partial E_T}{\partial x_j} = C(Q^2 \hat{k}^2)^{\frac{1}{4}} x^{-\hat{\epsilon}} \exp\left(-\frac{\{|\hat{\lambda}'| \log(x_j^2/x) + \hat{T}\}^2}{2\hat{\lambda}'' \log(1/x)} \right), \tag{2.32}$$

which may be compared with result (2.18) obtained by the simplified analytic treatment. Here $\hat{T} = \log(Q^2/\hat{k}^2)$. We see we have recovered the same general structure as before, that is a Gaussian E_T distribution as a function of $\log x_j$, with the characteristics (i) to (iv) listed in Sec. II A. The quantities are now more precisely determined. First the parameter $\hat{\epsilon}$, which controls the growth of the distribution with decreasing x, is given by

$$\hat{\epsilon} = \tilde{K}(\tfrac{1}{4}) - \tilde{K}(\tfrac{1}{2}) = 2\bar{\alpha}_s \log 2, \tag{2.33}$$

which would reduce to the previous result, $\epsilon = \lambda''/32$, if the quadratic form Eq. (2.10) of $\tilde{K}(\omega)$ were valid to $\omega = \frac{1}{4}$. The parameters $\hat{\lambda}'$ and $\hat{\lambda}''$ denote the derivatives of $\tilde{K}$ at $\omega = \frac{1}{4}$

$$\hat{\lambda}' \equiv \tilde{K}'(\tfrac{1}{4}), \qquad \hat{\lambda}'' \equiv \tilde{K}''(\tfrac{1}{4}), \tag{2.34}$$

and finally the normalization

$$C = \tilde{H}(0) \frac{(\lambda'')^{\frac{1}{2}} \bar{\mathcal{F}}_2^{(0)}(\tfrac{1}{4}) \bar{f}_0(\tfrac{1}{4})}{(\hat{\lambda}'')^{\frac{1}{2}} \bar{\mathcal{F}}_2^{(0)}(\tfrac{1}{2}) \bar{f}_0(\tfrac{1}{2})} \tag{2.35}$$

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where

$$\tilde{H}(0) \;=\; \bar{\alpha}_s \frac{\Gamma^4(\frac{1}{4})}{2\pi\Gamma^2(\frac{1}{2})} \;\approx\; 8.75\bar{\alpha}_s. \tag{2.36}$$

As before we can translate the Gaussian E_T distribution in $\log x_j$ to one in terms of rapidity y. Again the distribution turns out to have the form of Eq. (2.32),

$$\frac{\partial E_T}{\partial y} \;=\; C(Q^2\hat{k}^2)^{\frac{1}{4}} x^{-\hat{\epsilon}} \exp\left(\frac{-\{|\hat{\lambda}'|2y + \hat{T} + \frac{1}{2}\hat{\lambda}'^2 \log(1/x) - \frac{1}{2}|\hat{\lambda}'|\hat{T}\}^2}{2\hat{\lambda}'' \log(1/x)} \right), \tag{2.37}$$

but we see that there are additional, non-leading, terms in the exponential.

III. E_T FLOW WITH STRONG K_T ORDERING IMPOSED

We compare the BFKL forms of the E_T distribution, Eq. (2.18) and Eq. (2.32), with that obtained from conventional dynamics based on Altarelli-Parisi evolution in the small x limit. To make a strict comparison we take fixed α_s. Thus rather than inserting into Eq. (2.14) the BFKL solutions (2.8) and (2.9), we replace them by the double leading logarithm (DLL) forms

$$zg(z, k^2) \;\sim\; \exp\left(2\left[\bar{\alpha}_s \log\left(\frac{k^2}{Q_0^2}\right) \log\frac{1}{z} \right]^{\frac{1}{2}} \right) \tag{3.1}$$

$$\mathcal{F}(z, k^2, Q^2) \;\sim\; k^2 \exp\left(2\left[\bar{\alpha}_s \log\left(\frac{Q^2}{k^2}\right) \log\frac{1}{z} \right]^{\frac{1}{2}} \right). \tag{3.2}$$

We substitute these forms into Eq. (2.14) and Eq. (2.13), and change the k_j^2 integration variable to $t = \log(k_j^2/Q_0^2)$. We then obtain

$$x_j \frac{\partial E_T}{\partial x_j} \;\sim\; \frac{\bar{\alpha}_s}{F_2}(Q_0^2)^{\frac{1}{2}} \int_0^T dt \, \exp\left(\frac{t}{2} + 2\left[\bar{\alpha}_s(T-t)\log\left(\frac{x_j}{x}\right) \right]^{\frac{1}{2}} + 2\left[\bar{\alpha}_s t \log\frac{1}{x_j} \right]^{\frac{1}{2}} \right) \tag{3.3}$$

where $T \equiv \log(Q^2/Q_0^2)$.

It is informative to evaluate the E_T distribution for $x_j = \sqrt{x}$, that is for

$$\log\frac{1}{x_j^2} \;=\; \log\frac{1}{x} \;\equiv\; Y; \tag{3.4}$$

a point "midway" between the current jet and the proton remnants. This choice facilitates an analytic study and allows us to gain insight into the x and Q^2 dependence of the E_T flow. When $\log(x_j/x) = \log(1/x_j)$ we can evaluate Eq. (3.3) using saddle point methods. We find

$$x_j \frac{\partial E_T}{\partial x_j}\bigg|_{x_j=\sqrt{x}} \;\sim\; \bar{\alpha}_s(Q_0^2)^{\frac{1}{2}} \exp\left(\tfrac{1}{2}\beta T + \sqrt{2\bar{\alpha}_s TY}\left[\sqrt{\beta} + \sqrt{1-\beta} - \sqrt{2} \right] \right) \tag{3.5}$$

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where β is a known function of the ratio $T/2\bar{\alpha}_s Y$. To be precise

$$\beta = \tfrac{1}{2}\left(1 + \left\{1 - 4\left[1 + \sqrt{1 + T/2\bar{\alpha}_s Y}\right]^{-2}\right\}^{\frac{1}{2}}\right). \tag{3.6}$$

The $-\sqrt{2}$ in the square brackets in Eq. (3.5) arises from substituting

$$F_2(x, Q^2) \sim \exp\left(2\left[\bar{\alpha}_s \log\left(\frac{Q^2}{Q_0^2}\right) \log\left(\frac{1}{x}\right)\right]^{\frac{1}{2}}\right) \tag{3.7}$$

into Eq. (3.3), rather than using Eq. (2.20). After some algebra, it is possible to show that the approximate estimate of the integral in Eq. (3.3) predicts a slow increase of E_T with decreasing x_j.

We inspect the properties of the E_T flow, Eq. (3.5), in two different limits. First in the double leading logarithm limit, where $\alpha_s TY \gg 1$ but $\alpha_s T \sim 1$ and $\alpha_s Y \sim 1$, we see that $\beta \to 1$, and so

$$\left. x_j \frac{\partial E_T}{\partial x_j} \right|_{x_j = \sqrt{x}} \sim (Q^2)^{\frac{1}{2}} \exp\left(-(\sqrt{2} - 1)\sqrt{2\bar{\alpha}_2 TY}\right). \tag{3.8}$$

Thus in the DLL limit we find that the E_T distribution grows essentially as $(Q^2)^{\frac{1}{2}}$ with increasing Q^2. In the second limit, $1/x \to \infty$ at finite Q^2, we see that $\beta \to \frac{1}{2}$. Thus we find

$$\left. x_j \frac{\partial E_T}{\partial x_j} \right|_{x_j = \sqrt{x}} \sim e^{T/4} \sim (Q^2 Q_0^2)^{\frac{1}{4}}. \tag{3.9}$$

Moreover it is possible to show from Eq. (3.5) that with decreasing x the distribution decreases towards its final limit, which is indicated by Eq. (3.9) (modulo slowly varying logarithmic factors). This should be contrasted with the BFKL behaviour of Eq. (2.32) which increases as $x^{-\hat{\epsilon}}$ with decreasing x.

IV. CALCULATION OF E_T FLOW

Above we have obtained analytic forms for the leading behaviour of the E_T distribution in the $x_j \to 0$, $x/x_j \to 0$ limit, for fixed α_s. These expose characteristic features of the expected E_T flow in deep-inelastic scattering in the central region between, but well away from, the current jet and proton remnants. To obtain a more realistic estimate we solve the appropriate BFKL equations numerically, and use running α_s. From Eqs (1.3) and (2.13) we see that the E_T flow, for a given x, Q^2, can be expressed as

$$x_j \frac{\partial E_T}{\partial x_j} = \frac{1}{F_2} \int \frac{dk_p^2}{k_p^2} \int \frac{dk_\gamma^2}{k_\gamma^2} \left(\frac{3\alpha_s}{\pi}\right) \mathcal{F}_2\left(\frac{x}{x_j}, k_\gamma^2, Q^2\right) f(x_j, k_p^2) \int \frac{d\phi/2\pi}{\sqrt{k_p^2 + k_\gamma^2 + 2k_p k_\gamma \cos\phi}}. \tag{4.1}$$

In this paper we focus on the central region with $x_j < 10^{-2}$ and $x/x_j < 10^{-1}$. Thus to calculate the E_T flow we have to solve three different BFKL equations, one for f which

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resums the lower gluon ladders of Fig. 2(b), one for $\mathcal{F}_2$ which resums the upper gluon ladders, and finally one for $F_2(x, Q^2)$. To be precise we determine $f(x_j, k_p^2)$ for $x_j < 10^{-2}$ by solving the BFKL equation, as described in Ref. [13], starting from a gluon distribution at $x_j = 10^{-2}$ obtained from the MRS set of partons [15]. The quantity $\mathcal{F}_2/k_\gamma^2$ may be identified with the structure function of a gluon of (approximate) virtuality k_γ^2, integrated over its longitudinal momentum. The function $\mathcal{F}_2(z, k_\gamma^2, Q^2)$ is determined for $z < 10^{-1}$ by solving the BFKL equation, as described in Ref. [16], starting from the boundary condition $\mathcal{F}_2 = \mathcal{F}_2^{(0)}$ at $z = 10^{-1}$, where $\mathcal{F}_2^{(0)}$ is the contribution resulting just from the quark box (and crossed box) in Fig. 2(b). Finally $F_2(x, Q^2)$ is determined by the convolution of the gluon ladder with the quark box (and crossed box), which symbolically may be written $F_2 = f \otimes \mathcal{F}_2^{(0)}$, as described in [4].

The infrared cut-off on the transverse momenta integrations is taken to be $k_0^2 = 1 \text{ GeV}^2$ throughout, a value which makes the calculated values of F_2 consistent with the recent observations at HERA [2,3]. The predictions for the E_T flow are less sensitive to the choice of the cut-off than those of F_2. The ultraviolet cut-off on the integrations over the transverse momenta is taken to be 10^4 GeV^2; the results for $f, \mathcal{F}_2$ and F_2 are not sensitive to reasonable variations about this value. We shall see below, however, that the E_T-weighted integration of Eq. (4.1) is sensitive to the ultraviolet cut-off as x_j decreases towards the current jet.

Numerical results for the E_T flow are presented in Fig. 3 for $Q^2 = 10 \text{ GeV}^2$ and values of $x = 10^{-8}, 10^{-7}, ... 10^{-4}$. We show results for such low values of x so as to establish a sufficiently large central region in order to compare the general features of the distribution with the expectations of the leading order analytic treatment of Sec. II From Fig. 3 we indeed see a broad Gaussian shape in $\log x_j$, with a peak below $x_j = \sqrt{x}$ and a width which grows with decreasing x. Moreover the full numerical treatment shows an *increase* of the height of the distribution with decreasing x, but not as rapid as anticipated by the leading order analytic result.

We also show in Fig. 3 the E_T distributions obtained using Altarelli-Parisi (AP) evolution for two values of x. These confirm the expectation that they *decrease* with decreasing x. We come back to the BFKL comparison with AP when discussing Fig. 5, but first in Fig. 4 we explore the sensitivity to the choice of the ultraviolet cut-off in Eq. (4.1).

Having established the general features of the E_T flow, we focus on the distribution for $x = 10^{-4}$, a value which is accessible at HERA. In Fig. 4 the sequence of dashed curves show the effect of choosing the ultraviolet cut-off in Eq. (4.1) to be $k_{uv}^2 = 10^2, 10^3, ... 10^7 \text{ GeV}^2$, and the continuous curve is the result obtained if the cut-off is chosen to be Q^2/z as implied by energy-momentum conservation [17], where $z = x/x_j$. At relatively low values of Q^2, such as $Q^2 = 10 \text{ GeV}^2$, which are relevant to experiments at HERA, energy conservation has a significant effect on the E_T flow arising from BFKL gluon emission. Fig. 5 compares the E_T flow for $Q^2 = 10 \text{ GeV}^2$ with that for $Q^2 = 100 \text{ GeV}^2$ with, in each case, $k_{uv}^2 = Q^2/z$. Also shown are the expectations from Altarelli-Parisi evolution. As anticipated we see a significant difference between the E_T flow arising from BFKL and AP gluon emission, particularly at lower Q^2 values.

We may use Eq. (2.21) to translate the E_T flow as a function x_j into a distribution in terms of the rapidity y in the virtual-photon-proton c.m. frame. For a reasonably flat E_T distribution a good estimate of y is obtained if we insert the average value of k_j^2 into Eq. (2.21). For $Q^2 = 10 \text{ GeV}^2$ we see that $E_T \approx 2$ GeV per unit of rapidity and so we take

$k_j^2 = 4 \text{ GeV}^2$. The result for y is the scale shown above Fig. 5. If the E_T distribution observed by the H1 collaboration [18] is compared with Monte Carlo expectations, then reasonable agreement is found for $x > 10^{-3}$, but for $x < 10^{-3}$ more E_T appears to be present in the region between the current jet and the proton remnants than is implied by the favoured Monte Carlo calculations (which at present do not allow for BFKL emissions). The deficiency is most evident in the rapidity region $x_j \gtrsim 10^{-2}$, which extends beyond the central region shown in Fig. 5. However it is encouraging that a simple extrapolation of the BFKL curves of Fig. 5 into this region indicates a sizeable enhancement of the predicted E_T distribution over that expected from conventional AP dynamics.

V. CONCLUSIONS

Our main objective is to find characteristics which are unique to BFKL dynamics at small x. One striking feature of BFKL dynamics is the very strong $x^{-\lambda}$, with $\lambda \approx 0.5$, increase of the deep-inelastic structure function $F_2(x, Q^2)$ as x decreases in the HERA regime [4,19]. However this increase can be mimicked by conventional Altarelli-Parisi evolution from a very low starting scale $Q_0^2 = 0.3 \text{ GeV}^2$ [20]. Global or inclusive quantities like structure functions do not provide conclusive tests of BFKL dynamics and it has been advocated that the BFKL effect can, in principle, be more definitively identified by studies of the final state in deep-inelastic scattering. One possibility is Mueller's proposal [21] of measuring the x/x_j dependence of deep-inelastic events containing a forward-going identified jet of longitudinal momentum $x_j p$. Another, which we have studied here, is to measure the E_T flow accompanying deep-inelastic events at small x. A distinctive feature of BFKL dynamics is the diffusion pattern of the transverse momentum flow, in contrast to the strong-ordering of k_T towards Q^2 which is characteristic of conventional Altarelli-Parisi dynamics. Indeed it is the absence of strong k_T-ordering which leads to a strongly enhanced E_T distribution at small x. We analysed this effect both analytically and numerically, focussing attention on the central region between the current jet and the proton remnants.

In Sec. III we performed a detailed analytic analysis of the E_T flow. We found that BFKL dynamics with fixed α_s gives a broad Gaussian E_T distribution as a function of rapidity (or rather $\log x_j$) which grows as $x^{-\epsilon}$ with decreasing x where $\epsilon = (3\alpha_s/\pi)2\log 2$. This should be contrasted with the much smaller E_T flow obtained assuming strong k_T-ordering, which gives an E_T distribution that decreases with decreasing x, for fixed Q^2. In Section 4 we confirmed these qualitative features using numerical solutions of the appropriate BFKL equations. As expected we found that the E_T flow which accompanies deep-inelastic events is much larger for BFKL than for conventional AP dynamics. We also showed, at the low Q^2 values relevant to HERA, that energy conservation should be used to limit the energy-weighted integrations over transverse momentum.

Taking, for example, deep-inelastic events with $x = 10^{-4}$ and $Q^2 = 10 \text{ GeV}^2$ we found that the E_T flow in the central rapidity region ($x_j < 10^{-2}$, $x/x_j < 10^{-1}$) between the current jet and the proton remnants is about 2 GeV per unit of rapidity. This result is very encouraging for the identification of BFKL dynamics. However for a quantitative comparison with experiment we will need to include contributions to the E_T flow coming from the hadronization of the produced partons, not only from the BFKL ladder, but also from the partons radiated from the current jet. Also there is a contribution radiated from

the colour string which connects the current jet to the proton remnants. An analysis of this problem is under study and its results will be reported elsewhere.

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FIGURES

FIG. 1.   Deep-inelastic scattering at small x, accompanied by soft gluon emission with total transverse energy ΣE_{Ti}.

FIG. 2.   (a) Diagrammatic representation of Eq. (1.3); (b) explicit display of the gluon ladders which are resummed by the BFKL equations for f and $\mathcal{F}_i$.

FIG. 3.   The E_T flow as a function of $\log x_j$, calculated using BFKL forms in Eq. (4.1), for $x = 10^{-8}, 10^{-7}, ...10^{-4}$ and $Q^2 = 10$ GeV^2, in the central region ($x_j < 10^{-2}$, $x/x_j < 10^{-1}$) between the current jet ($x_j \approx x$) and the proton remnants ($x_j \approx 1$). For comparison, the dashed curves show the E_T flow calculated using Altarelli-Parisi evolution for $x = 10^{-6}$ and 10^{-4}.

FIG. 4.   The sensitivity of the E_T flow (for $x = 10^{-4}$ and $Q^2 = 10$ GeV^2) to the choice of the ultraviolet cut-off k_{uv}^2 imposed on the integrations over the transverse momenta in Eq. (4.1). The dashed curves correspond to $k_{\mathrm{uv}}^2 = 10^7, 10^6, ...10^2$ GeV^2, and the continuous curve to $k_{\mathrm{uv}}^2 = Q^2/z$ with $z = x/x_j$.

FIG. 5.   The E_T flow for $x = 10^{-4}$ calculated using BFKL forms (continuous curves) and conventional AP dynamics (dashed curves) for two different values of Q^2, namely $Q^2 = 10$ and 100 GeV^2. The BFKL predictions are obtained from Eq. (4.1) with $k_{\mathrm{uv}}^2 = Q^2/z$ with $z = x/x_j$. The variable $\log x_j$ is related to the virtual-photon-proton c.m. rapidity, y, via Eq. (2.21). The approximate rapidity scale for $Q^2 = 10$ GeV^2 is shown above the figure.

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